1 Definitions

1. A simple extension field $K(\alpha) : K$ is called algebraic if α is the zero of a polynomial in $K[x]$.
2. Otherwise, a simple extension field $K(\alpha) : K$ is called transcendental.
3. The degree of a simple extension $F(\alpha) : F$ is the degree of the minimum polynomial of α over F.
4. A vector space V over a field K is a set V such that
   (a) V is a group under $+$.
   (b) $k \cdot v \in V$ if $k \in K, v \in V$.
   (c) $(k \times l) \cdot v = k \cdot (l \cdot v), k, l \in K, v \in V$.
   (d) $(k + l) \cdot v = k \cdot v + l \cdot v, k, l \in K, v \in V$.
   (e) $k(u + v) = k \cdot u + k \cdot v, k \in K, u, v \in V$.
   (f) $1v = v, 1$ is unity, $v \in V$.
5. A basis is a linearly independent spanning set.
6. A spanning set is a set such that every vector in the vector space is a linear combination of the elements of the spanning set.
7. The dimension of a vector space is the number of elements in its basis.
8. The dimension of E over F is denoted $[E : F]$.
9. If α is algebraic over K, then $[K(\alpha) : K]$ is the degree of the minimum polynomial of α.

2 Theorem

Theorem 1. *If K is a field, then there is only one transcendental simple extension (up to isomorphism).*

Theorem 2. *If $K(\alpha)$ is an algebraic extension, then there exists a unique monic irreducible polynomial of minimum degree, $p(x)$, such that α is a zero of $p(x)$.*

Corollary 3. *If $m(\alpha) = 0$, then the minimum polynomial $p(x)$ will divide m.*

Theorem 4. *If $K(\alpha)$ is an algebraic extension of K and $p(x)$ is the minimum polynomial of α, then any element in $K(\alpha)$ can be written as a polynomial in α of degree less than the degree of $p(x)$.*

Theorem 5. *If α is algebraic over K, then $K(\alpha) = K[\alpha]$.*

Theorem 6. *If $m(x)$, a polynomial of degree greater than 0, is irreducible in $K[x]$, then $K[x]/ < m(x) >$ is a field.*

Theorem 7. *If K is a field and m a monic irreducible polynomial, then there exists $K(\alpha) : K$ such that α is a zero of m.*

Theorem 8. *If E is an extension of F, then E is a vector space.*