Notebook Problem 5.6.34

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Theorem. *Let $f(x) \in F[x]$ where F is a field, and let $\alpha \in F$. Then the remainder $r(x)$ when $f(x)$ is divided by $x - \alpha$ is $f(\alpha)$.*

Proof. First, if $x - \alpha$ divides $f(x)$, then α is a zero of $f(x)$, so $f(\alpha) = 0$, and the remainder $r(x)$ when $f(x)$ is divided by $x - \alpha$ is also 0.

Now, if $x - \alpha$ does not divide $f(x)$, the remainder $r(x)$ must have degree less than the degree of $x - \alpha$ (by the Division Algorithm), so $r(x)$ must be a constant term. Call it k. Thus, $f(x) = (x - \alpha)q(x) + k$ and $f(\alpha) = (\alpha - \alpha)q(\alpha) + k$, so that $f(\alpha) = k = r(x)$. $\square$