Notebook problem #1

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Theorem. *For every* $n \in \mathbb{Z}$*, there is exactly one ring with unity whose group under its first operation is* $< \mathbb{Z}_n, + >$.

Proof. Suppose A, B are rings with unity such that $|A| = |B| = n$ and $< A, + > \cong < B, + > \cong < \mathbb{Z}_n, + >$, but $A \not\cong B$. Let $a_k \cong b_k \cong k$ where a_k and b_k are considered as elements of the groups $< A, + >$ and $< B, + >$, respectively.

Let a_1 be the multiplicative identity in the ring A and b_m be the multiplicatve identity in ring B. If $m = 1$, then, since $a_1 \cong b_1$ to keep the addition isomorphism, we have that the multiplicative identities are isomorphic, so the rings themselves must be isomorphic.

So now we suppose that $m \neq 1$. However, $a_1 a_c = a_c \cong b_c = b_m b_c$, so that $a_1 a_c \cong b_m b_c$, which gives that $a_1 \cong b_m$, so the identities are isomorphic, so $A \cong B$. □