

# Notebook Problem Fraleigh 9.4.18

Jason Wojciechowski

25th April 2002

**Theorem.** *Every irreducible polynomial over a field  $F$  of characteristic 0 is separable.*

*Proof.* Fraleigh's Theorem 9.4.13 gives that every field of characteristic 0 is perfect, so every finite extension of a field  $F$  of characteristic 0 is separable. A polynomial is separable over  $F$  if all of its zeros are separable, and the zero of a polynomial, say  $\alpha$ , is separable if  $F(\alpha)$  is a separable extension.

Thus  $F(\alpha)$  is separable in any field of characteristic 0, so  $\alpha$  is separable, so  $\alpha$ 's minimum polynomial, which is an irreducible polynomial, is separable.  $\square$