

Notebook Problem Fraleigh 9.4.18

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Theorem 1. *Every irreducible polynomial over a field F of characteristic 0 is separable.*

Proof. The irreducible polynomial f for an arbitrary $\alpha \in \bar{F}$ over some field F is also the polynomial of minimum degree for α , so f' , the derivative of f , which has degree one less than f , does not have α as a zero unless f' is the zero polynomial. However, there are only two ways f' could be the zero polynomial: if f was a constant polynomial or if the coefficients of f' were all zeros. We know f is non-constant because constant polynomials are not irreducible. We know the coefficients of f' are not zero because they are all of the form $a_i(i \cdot 1)$. There is at least one a_i which is non-zero, or else we would have the zero polynomial, and $i \cdot 1$ is not zero because F has characteristic 0.

Thus, by exercise 17, α is a zero of multiplicity one, so all of f 's zeros are of multiplicity one, so f is separable. \square